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Resonance Models for Nucleon Electromagnetic Structure*

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Theoretical restrictions that must be imposed on models of nucleon electromagnetic structure are derived. The equality of electric and magnetic form factors at the threshold for nucleon-antinucleon annihilation ($q^2=-4M^2$) is established from the proper momentum dependence of the s- and d-wave matrix elements of the electromagnetic current density. The role of this equality in approximate theoretical treatments of G_E and G_M is discussed. Certain general implications of recent elastic electron-proton scattering data are interpreted as additional constraints to be imposed on resonance models which have no "core contributions." One such proposed four-pole model involving the $[\omega,\phi,\rho,B]$ vector resonances is found to be inconsistent with certain of these restrictions.

I. INTRODUCTION

Results of recent cross-section measurements for elastic electron-proton scattering at high momentum transfer[1] have invited renewed speculation on theoretical models for nucleon electromagnetic structure.[2,3] These new data appear consistent with an asymptotic decrease proportional to $1/q^2$ for both the electric and magnetic proton form factors. Such behavior suggests the attractive possibility of unsubtracted dispersion relations with spectral functions that are dominated by p-wave multimeson resonances of low mass.[4] Our intent is to examine some restrictions that must be imposed on these models of the nucleon electromagnetic form factors in addition to the well-known boundary values at zero momentum transfer.

In Sec. II the Dirac and Pauli form factors are shown to be free of kinematic singularities; this fact alone implies the equality of the electric and magnetic form factors at the threshold of the nucleon-antinucleon annihilation channel. A sufficient condition to derive this threshold equality is the proper momentum dependence of the s- and d-wave matrix elements of the electromagnetic current density at the threshold for $N\bar{N}$ annihilation through a virtual photon.

In Sec. III we are primarily concerned with the proposed resonance fits for the form factors. The equality of G_E and G_M at $q^2=-4M^2$ and certain other general implications of the experimental data are interpreted as restrictions on the parameters for such models.

II. THRESHOLD BEHAVIOR IN THE $N\bar{N}$ ANNIHILATION CHANNEL

The conventional decomposition of the matrix element of the nucleon current into Dirac and Pauli form factors is[5]

$$\langle p'|J_\mu|p\rangle = i(M^2/p_0'p_0)^{1/2}\bar{u}(\mathbf{p}') \times[\gamma_\mu F_1(q^2)-\sigma_{\mu\nu}(q_\nu/2M)F_2(q^2)]u(\mathbf{p}). \quad (1)$$

The electric and magnetic form factors, which have direct physical interpretation as the distribution of charge and magnetization in the nucleon,[6,7] are related to the Dirac and Pauli form factors by

$$G_E(q^2)=F_1(q^2)-(q^2/4M^2)F_2(q^2),$$
$$G_M(q^2)=F_1(q^2)+F_2(q^2). \quad (2)$$

Once we can establish that F_1 and F_2 are nonsingular at the threshold of the nucleon-antinucleon channel, the equality

$$G_E(-4M^2)=G_M(-4M^2) \quad (3)$$

follows at once from the defining equations, Eq. (2). This threshold condition holds of course for both proton and neutron (or, alternatively, for both vector and scalar) form factors.[8]

The standard procedure for showing that the invariant amplitudes are free of kinematic singularities[9,10] breaks down precisely at the point of interest, $q^2=-4M^2$. To see this, we consider the quantity

$$\Gamma_\mu=(M-i\gamma\cdot p')[\gamma_\mu F_1(q^2)-\sigma_{\mu\nu}(q_\nu/2M)F_2(q^2)] \times(M-i\gamma\cdot p), \quad (4)$$

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[1] K. W. Chen, A. A. Cone, J. R. Dunning, Jr., S. G. F. Frank, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Letters **11**, 561 (1963).

[2] R. G. Sachs, Phys. Rev. Letters **12**, 231 (1964).

[3] A. P. Balachandran, P. G. O. Freund, and C. R. Schumacher, Phys. Rev. Letters **12**, 209 (1964).

[4] Before experimental data at high momentum transfer was available, a number of authors considered resonance models with core contributions. See, for example, E. Clementel and C. Villi, Nuovo Cimento **4**, 1207 (1956); R. Hofstadter and R. Herman, Phys. Rev. Letters **6**, 293 (1961); S. Bergia, A. Stanghellini, S. Fubini, and C. Villi, Phys. Rev. Letters **6**, 367 (1961); K. C. Wali, Nuovo Cimento **25**, 912 (1962).

[5] The F_j are still operators in isospin space, $F_j=F_j^S+\tau_3F_j^V$ ($j=1,2$) and so $F_j^p=F_j^S+F_j^V$, $F_j^n=F_j^S-F_j^V$.

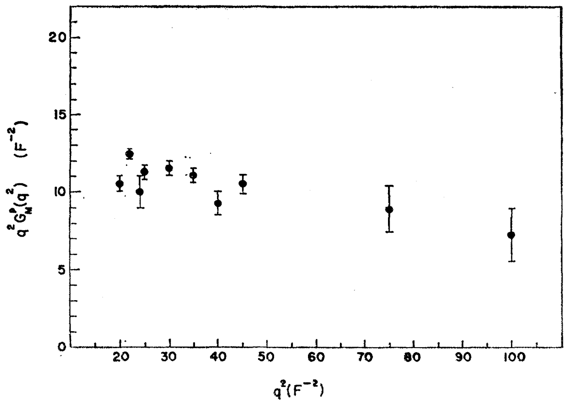
[6] F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960).

[7] R. G. Sachs, Phys. Rev. **126**, 2256 (1962).

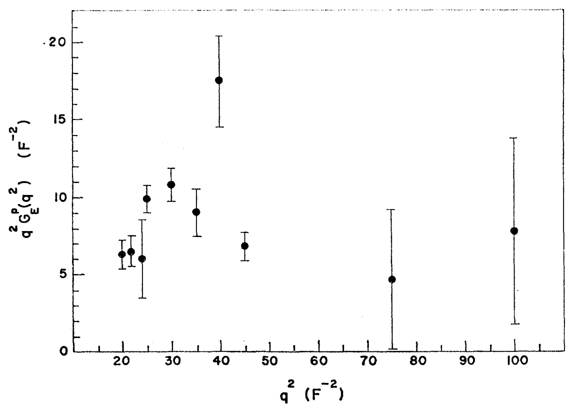
[8] S. Bergia and L. Brown, Stanford Conference, 1963, (unpublished) have also considered this threshold equality [L. Brown (private communication)].

[9] A. M. Bincer, Phys. Rev. **118**, 855 (1960).

[10] J. S. Ball, Phys. Rev. **124**, 2014 (1961).



(a)



(b)

FIG. 1. Experimental data on the proton electric and magnetic form factors for 4-momentum transfers ranging from 20 F^{-2} to 100 F^{-2}: (a) $q^2G_M{}^p(q^2)$, (b) $q^2G_E{}^p(q^2)$.

which is analytic except for the usual dynamical cut. Thus the traces over the spin indices

$$T_1(q^2)=\mathrm{Tr}\{\gamma_\mu\Gamma_\mu\},\qquad T_2(q^2)=(q_\nu/2M)\,\mathrm{Tr}\{\sigma_{\mu\nu}\Gamma_\mu\},\tag{5}$$

yield functions of q^2 with no more singularities than those required by unitarity. Solving Eqs. (4) and (5) for F_1 and F_2, we find

$$F_1(q^2)=q^2[(q^2/2M^2-4)T_1(q^2)+6T_2(q^2)]D^{-1}(q^2),$$
$$F_2(q^2)=[6q^2T_1(q^2)+4(2M^2-q^2)T_2(q^2)]D^{-1}(q^2),\tag{6}$$

where $D(q^2)=-(2q^2/M^2)(q^2+4M^2)^2$. The absence of kinematic singularities in the T_i implies the absence of such singularities in the F_i except possibly at $q^2=0$ for F_2 and $q^2=-4M^2$ for F_1 or F_2. Since each of these points is at the boundary of a physical region, arguments based on the known behavior of *physical* matrix elements can be invoked to establish regularity. At $q^2=0$ the finiteness of the nucleon magnetic moments implies the finiteness of F_2 there. At $q^2=-4M^2$ regularity of the F_1 and F_2 follows from consideration of the $N\bar{N}$ annihilation through a virtual photon: We examine the matrix element of the current in the rest frame of the nucleon-antinucleon pair. The annihilation occurs only in the $^3S_1$ and $^3D_1$ pair states. For small nucleon momenta $|\mathbf{p}|$, the matrix element has the form[11]

$$\langle 0|\mathbf{J}|p\bar{p}\rangle=-\chi_{\bar{p}}{}^\dagger[(F_1+F_2)\boldsymbol{\sigma}+(\mathbf{p}^2/6M)\times(F_2-F_1)(3\boldsymbol{\sigma}\cdot\hat{p}\hat{p}-\boldsymbol{\sigma})]\chi_p.\tag{7}$$

In order that the individual partial waves of this matrix element have the required $|\mathbf{p}|^{2l}$ behavior near threshold, both (F_1+F_2) and (F_1-F_2) must be finite as $|\mathbf{p}|\to 0$. This eliminates the possibility of a kinematic singularity in the F_i at $q^2=-4M^2$. Of course in the absence of experimental information we cannot actually say that the $|\mathbf{p}|^{2l}$ threshold behavior is obeyed. We ignore the unlikely occurrence of a dynamical pole at $q^2=-4M^2$ which would alter the normal threshold momentum dependence. Such a dynamical singularity would correspond to the presence of a stable vector meson of mass $2M$ and would lead to a greatly enhanced $N\bar{N}\to e\bar{e}$ cross section near threshold.

The question naturally arises to what extent Eq. (3) should be maintained in an approximate treatment of the form factors; in particular, we have in mind structural models based on the dominance of resonant p-wave intermediate states. For the following two reasons we feel that this threshold equality must be maintained in any approximate theoretical treatment of the electric and magnetic form factors: First, failure to impose this condition necessarily leads to a pole term in F_i of the form $\alpha_i/(q^2+4M^2)$. In addition to destroying the proper analytic structure of the form factors, this extraneous pole vitiates the interpretation of the residues of legitimate poles as coupling constants and, in fact, may well obscure the theoretical significance of empirical fits to the experimental data. Secondly, even in the more limited focus where nearby singularities and thresholds are assumed predominant, the $N\bar{N}$ threshold is not much higher than the masses of resonances commonly assumed to be of importance in the form factors. For example, the mass of the ϕ meson is 1020 MeV and the mass of the B meson, which may also be a vector resonance,[12] is 1220 MeV as compared with 1878 MeV for the $N\bar{N}$ threshold. Thus, we conclude that meaningful theoretical approximations to the real nucleon electromagnetic structure should embody the equality of G_E and G_M at $q^2=-4M^2$ to a good approximation.

III. RESONANCE MODELS

The experimental data on the proton electric and magnetic form factors[1,13] are plotted in Fig. 1 for 4-momentum transfers ranging from 20 F^{-2} to 100 F^{-2}.

[11] P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. 112, 642 (1958).
[12] W. R. Frazer, S. H. Patil, and N. Xuong, Phys. Rev. Letters 12, 178 (1964).
[13] L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).

These data have been interpreted[1,3] to be consistent with the assumption that

$$\lim_{q^2\to\infty} q^2 G_M{}^p(q^2) = C_M{}^p\,,$$
$$\lim_{q^2\to\infty} q^2 G_E{}^p(q^2) = C_E{}^p\,, \tag{8}$$

where $C_M{}^p$ and $C_E{}^p$ are constants. If we accept this interpretation of the data at face value, then with equal validity we infer from the apparent continuity of the data that

$$C_M{}^p \geqslant 0\,,$$
$$C_E{}^p \geqslant 0\,. \tag{9}$$

(Note that noninterference of G_E and G_M in the Rosenbluth formula makes direct determination of the sign of either form factor impossible in electron-nucleon scattering experiments.[13] However, interference does occur in electron-deuteron scattering.[14])

The physical interpretation of Eqs. (8) and (9) is that the charge and current densities of the proton in configuration space remain positive as the origin of the distribution is approached. This interpretation follows from the relationship of the charge and current densities to the Fourier transform of the form factors evaluated in the Breit frame,[7] i.e., $q^2=\mathbf{q}^2$.

$$\rho(\mathbf{r}) = e(2\pi)^{-3}\int d^3q\, G_E(\mathbf{q}^2)e^{-i\mathbf{q}\cdot\mathbf{r}}\,,$$
$$\mathbf{J}(\mathbf{r}) = ie(2\pi)^{-3}\int d^3q(\boldsymbol{\sigma}\times\mathbf{q})G_M(\mathbf{q}^2)e^{-i\mathbf{q}\cdot\mathbf{r}}\,. \tag{10}$$

From Eqs. (8) and (10) we obtain the desired result for resonance models:

$$\rho_p(\mathbf{r}) \underset{r\to 0}{\longrightarrow} e(4\pi r)^{-1}C_E{}^p\,,$$
$$\mathbf{J}_p(\mathbf{r}) \underset{r\to 0}{\longrightarrow} e(4\pi r^3)^{-1}(\boldsymbol{\sigma}\times\mathbf{r})C_M{}^p\,. \tag{11}$$

Experimental information on asymptotic behavior exists only for the proton form factors. In the following treatment we shall also employ the reasonable conjecture that the same $1/q^2$ asymptotic decrease obtains for the neutron form factors, G^n. This is entirely consonant with the spirit of recent resonance models for electromagnetic nucleon structure.[3] At this point an examination of Eq. (2) and the inverse equations for the Dirac and Pauli form factors, along with the information on analytic properties of the form factors derived in Sec. II, shows the complete equivalence of the following two statements:

(i) The form factors G_E and G_M satisfy unsubtracted dispersion relations (no kinematic singularities) and

$$G_E(-4M^2) = G_M(-4M^2)\,.$$

[14] D. J. Drickey and L. N. Hand, Phys. Rev. Letters **9**, 521 (1962).

(ii) The form factors F_1 and F_2 satisfy unsubtracted dispersion relations (no kinematic singularities) and

$$\lim_{q^2\to\infty} q^2 F_2(q^2) = 0\,.$$

Although the electric and magnetic form factors are advantageous from the experimental point of view,[13] the F's are more useful for some theoretical analyses.

The simplest possible resonance model without "core contributions" consists of two isovector resonances (V, V') and two isoscalar resonances (S, S'). In this case the Dirac and Pauli form factors may be expressed as

$$F_1{}^i(q^2) = (1/2)[f_1{}^i/(1+q^2/i) - (f_1{}^i-1)/(1+q^2/i')]\,,$$
$$F_2{}^i(q^2) = \kappa_i[f_2{}^i/(1+q^2/i) - (f_2{}^i-1)/(1+q^2/i')]\,, \tag{12}$$
$$i = S(\text{scalar}) \text{ or } V(\text{vector})\,,$$

where the label i also denotes the (mass)^2 of the ith resonance, e.g., $S=M_S{}^2$, $S'=M_{S'}{}^2$. The known $F(0)$ values have already been imposed. From the asymptotic conditions $q^2F_2{}^i\to 0$, we determine the residues of the poles of $F_2{}^i$,

$$f_2{}^i = i'/(i'-i)\,,\quad i=S,\ V\,. \tag{13}$$

On the basis of certain physical arguments concerning the structure of the nucleon, Sachs has also suggested the limit on the proton form factor

$$\lim_{q^2\to\infty} G_E{}^p(q^2)/G_M{}^p(q^2) = 1 \tag{14}$$

as well as the asymptotic limits $q^2F_2{}^i\to 0$.[2,7] Furthermore, the recent Cambridge experiment is consistent with $G_E{}^p=G_M{}^p$ at large momentum transfers.[1] We are now in a position to investigate Eq. (14) within the framework of the resonance model. From Eq. (2) we see that the limit given in Eq. (14) can also be expressed as

$$\lim_{q^2\to\infty} (q^2)^2 F_2{}^p(q^2) = 0 \tag{15}$$

for noncore resonance models. In terms of the 4-pole structure, we obtain from Eqs. (12), (13), and (15) a condition on the masses of the contributing resonances, namely, that

$$(S'/V')(S/V) = -\kappa_V/\kappa_S = 30.9\,. \tag{16}$$

This relation is completely incompatible with reasonable choices for the masses. If, for example, we choose the phi and rho mesons for S and V, then we find $M_{S'} = 4.1M_{V'}$. This solution obviously violates the nearby singularity concept. Such large mass splitting also precludes the assignment of S' and V' to an octet representation of SU_3. Thus, it appears that the Sachs limit, Eq. (14), can never be realized in the 4-pole noncore resonance model. Therefore, we will not insist on this limit further but will go on to the other restrictions inferred from experiment.

An examination of experimental conditions which

TABLE I. Experimental derivatives of the form factors at zero momentum transfer.

$\frac{dG_E^p}{dq^2}(0) = -0.108 \pm 0.003$ F^{+2}
$\frac{dG_E^n}{dq^2}(0) = 0.021 \pm 0.001$ F^{+2}
$\frac{dG_M^p}{dq^2}(0) = -0.30 \pm 0.02$ F^{+2}
$\frac{dG_M^n}{dq^2}(0) = +0.20 \pm 0.08$ F^{+2}

the 4-pole resonance model must satisfy is more conveniently accomplished in terms of electric and magnetic form factors, as previously noted. The pole expansion incorporating the known $G(0)$ values may be written

$$G_E{}^i(q^2) = (1/2)[g_E{}^i/(1+q^2/i) - (g_E{}^i-1)/(1+q^2/i')],$$
$$G_M{}^i(q^2) = \mu_i[g_M{}^i/(1+q^2/i) - (g_M{}^i-1)/(1+q^2/i')], \quad (17)$$
$$i = S, V.$$

In demanding that the threshold equality in Eq. (3) hold for Eqs. (17), we are led to a relation between the residues of the poles in the electric and magnetic form factors:

$$g_M{}^i = (1/2\mu_i)[g_E{}^i + (2\mu_i - 1)(1 - i/4M^2)/(1 - i/i')], \quad i = S, V. \quad (18)$$

We recall that this relation is essential to the reciprocity of statements (i) and (ii). The connection between Eqs. (12) and Eqs. (17) may now be expressed in terms of the corresponding residues.

$$f_1{}^i = (g_E{}^i - i\mu_i g_M{}^i/2M^2)/(1 - i/4M^2),$$
$$f_2{}^i = (1/\kappa_i)(\mu_i g_M{}^i - g_E{}^i/2)/(1 - i/4M^2), \quad (19)$$
$$i = S, V.$$

In principle, the residues g^i are directly determinable from the experimental derivatives of the form factors and the resonance masses by the equations:

$$g_E{}^i = -\left[2i'\frac{dG_E{}^i}{dq^2}(0) + 1\right] i/(i'-i),$$
$$g_M{}^i = -\left[i'\frac{dG_M{}^i}{dq^2}(0)/\mu_i + 1\right] i/(i'-i), \quad (20)$$
$$i = S, V.$$

In practice, however, the $g_M{}^i$ are more accurately obtained from Eq. (18) since the experimental values of the electric derivatives are more precisely known than the magnetic (see Table I). The residues are now overdetermined by Eqs. (18) and (20) which results in a consistency requirement for the model (or, alternatively, a restriction on the range of resonance masses compatible with the experimental errors on the derivatives). Combining Eqs. (18) and (20) gives

$$1/i' + 1/i - 1/4M^2 + \left[\frac{dG_M{}^i}{dq^2}(0) - \frac{dG_E{}^i}{dq^2}(0)\right] \Big/ (\mu_i - 1/2) = 0, \quad (21)$$
$$i = S, V.$$

Finally, we examine the consequences of Eq. (9) for the 4-pole resonance model. A straightforward calculation yields the following constraints on the masses of contributing resonances:

$$\lim_{q^2\to\infty} (q^2 G_E{}^p) = \sum_{S,V} \left\{ i'\left[i\frac{dG_E{}^i}{dq^2}(0) + 1/2 \right] + i/2 \right\} \geqslant 0 \quad (22)$$

and

$$\lim_{q^2\to\infty} (q^2 G_M{}^p) = \sum_{S,V} \left\{ i'\left[i\frac{dG_M{}^i}{dq^2}(0) + 1/2 \right] + i/2 \right\} \geqslant 0. \quad (23)$$

Through the use of the threshold condition at $q^2 = -4M^2$, as expressed by Eq. (18), Eq. (23) can be converted to the more useful form

$$\lim_{q^2\to\infty} (q^2 G_M{}^p) = \sum_{S,V} \left\{ i'\left[i\frac{dG_E{}^i}{dq^2} + 1/2 + (\mu_i - 1/2) i/4M^2 \right] + i/2 \right\} \geqslant 0. \quad (24)$$

Since the magnitude of $\mu_S - 1/2 = -0.06$ is quite small compared to $\mu_V - 1/2 = 1.85$, the positivity condition on $\lim_{q^2\to\infty}(q^2 G_E{}^p)$ given in Eq. (22) is more stringent than the positivity condition on $\lim_{q^2\to\infty}(q^2 G_M{}^p)$ given in Eq. (24) above. Equations (21), (22), and (24) constitute basic conditions that must be satisfied if a 4-pole resonance structure is a meaningful approximation to nucleon electromagnetic structure. We now turn to numerical evaluation of these restrictions.

A 4-pole resonance model is interesting only if positive identifications can be made with experimental resonances. In a recent letter, Balachandran, Freund, and Schumacher[3] suggested that the isoscalar resonances $\omega(1^{--}, 783)$, $\phi(1^{--}, 1020)$ and the isovector resonances $\rho(1^{-+}, 750)$, $B(?, 1220)$ provide a satisfactory explanation of the observed nucleon electromagnetic structure. In our subsequent analysis, we apply the set of criteria established in the previous paragraphs to their proposal. The experimental derivatives of the form factors at zero momentum transfer are tabulated in Table I.[13] Using these values, along with the associated experimental uncertainties, the left-hand sides of Eqs. (21), (22), and (24) have been calculated for the appropriate masses: $S = 15.8$ F^{-2}, $S' = 26.7$ F^{-2}, $V = 14.5$ F^{-2}, $V' = 38.2$ F^{-2}. The results are recorded in Table II.

TABLE II. Numerical evaluation of criteria on the validity of the suggested $\{\omega,\phi,\rho,B\}$ resonance model for nucleon electromagnetic structure. The left-hand sides of Eqs. (21), (22), and (24) are numerically tabulated for this model.

Equation number	Numerical result	Theoretical constraint
(21) Vector case	(-0.016 ± 0.022) F^2	$=0$
(21) Scalar case	(0.199 ± 0.687) F^2	$=0$
(22) $\lim_{q^2\to\infty}(q^2G_E{}^p)$	(-6.36 ± 1.46) F^{-2}	$\geqslant 0$
(24) $\lim_{q^2\to\infty}(q^2G_M{}^p)$	(4.68 ± 1.46) F^{-2}	$\geqslant 0$

Within the experimental errors, the model is not inconsistent with the threshold condition, Eq. (21), and the positivity condition on $\lim_{q^2\to\infty}(q^2G_M{}^p)$, Eq. (24). However, the positivity restriction on $\lim_{q^2\to\infty}(q^2G_E{}^p)$, Eq. (22), is badly violated. It should be noted that this latter condition is completely independent of the threshold condition. Although the effective masses of the vector particles may be shifted slightly due to their broad widths,[15] this effect (or the experimental uncertainties in the masses) does not significantly modify the above results. Consequently, we conclude that the $\{\omega,\phi,\rho,B\}$ resonance model cannot accommodate all of the most evident general features of the experimental data on nucleon electromagnetic form factors.

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[15] M. W. Kirson, Phys. Rev. 132, 1249 (1963).

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Particle Mixing and Renormalization*

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The field theory of neutral vector particles interacting with conserved currents is investigated as an example of particle mixing. It is shown that a generalization of conventional renormalization is necessary when mixing occurs, and that the observable masses and coupling constants are sufficient to determine transition amplitudes, without recourse to mixing parameters. The universality of electric charge renormalization is not changed when photon–vector-meson mixing is possible.

INTRODUCTION

THEORETICAL and experimental physicists are currently investigating mixing between particles of the same spin, parity, charge, and baryon number.[1–4] It is the aim of this paper to show how a sound theoretical basis might be given, from which the consequences of mixing could be predicted. In the belief that it is the most interesting and physically relevant case, we confine the discussion to the mixing of neutral vector particles, such as the photon and φ, ρ, ω mesons, which is caused by their interactions with conserved currents (which we shall assume renormalizable).

In the main part of the paper we show how mixing may be correctly taken into account by an extension of conventional renormalization, and we then consider photon-vector-meson mixing as a particular case.

VECTOR-PARTICLE FIELD THEORY

We can use covariant notation[5] to write the Lagrangian density[6] for neutral vector fields $A_\nu{}^i$, $i=1,\cdots n$, interacting with conserved currents $J_\mu{}^\alpha$, $\alpha=1,\cdots,N$, in a particular but arbitrary gauge specified by constants λ_i. This method allows us to consider massive and massless particles together; for the latter, we shall put in a mass M, and take the limit $M\to 0$ at the appropriate place.

$$L=L_0[A^i,m_i,\lambda_i]-\sum_\alpha g_{i\alpha}A_\mu{}^iJ_\mu{}^\alpha + \text{terms not involving the } A\text{'s}, \quad (2.1)$$

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[1] G. Feldman and P. T. Matthews, Phys. Rev. 132, 823 (1963).

[2] J. B. Baroff and T. Fulton, Nuovo Cimento 29, 687 (1963).

[3] T. Kaneko, Y. Ohnuki, and K. Watanabe, Progr. Theoret. Phys. (Kyoto) 30, 521 (1963).

[4] S. Coleman and H. J. Schnitzer, Phys. Rev. 134, B863 (1964).

[5] See G. Feldman and P. T. Matthews, Phys. Rev. 130, 1633 (1963).

[6] We use the notation $a_\mu b_\mu\equiv a_0b_0-\mathbf{a}\cdot\mathbf{b}$ and $\partial_\mu\equiv(\partial/\partial x_0, -\partial/\partial\mathbf{x})$. Repeated indices i, j, k are summed over, but the repeated index α is only summed over when Σ_α precedes the expression involved.